Supplementary Material: Pilots and Other Predictable Elements of the Starlink Ku-Band Downlink

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I. Derivation of Processing Gain for OFDM Signals

This appendix provides the complete derivation of the processing gain expressions presented in Section III. We evaluate L as defined in (1) under three conditions for the information symbols $\{x_k\}_{k=1}^N$: (i) completely unknown, (ii) completely known, and (iii) partially known.

A. Unknown Data

Suppose the sequence $\{x_k\}_{k=1}^N$ is completely unknown. Then, without loss of generality, we set $l_k = 1$ for all $k \in [1, N]$. Since the x_k are zero-mean and independent,

$$\begin{aligned}\mathbb{E}\left[S_x^* S_x\right] &= \mathbb{E}\left[\left(\sum_{k=1}^N x_k^*\right)\left(\sum_{j=1}^N x_j\right)\right] \\ &= \sum_{k=1}^N \mathbb{E}\left[|x_k|^2\right] + \sum_{\substack{k,j=1 \\ k\neq j}}^N \mathbb{E}\left[x_k^*\right]\mathbb{E}\left[x_j\right] \\ &= N + 0 = N\end{aligned}$$

Similarly, $\mathbb{E}\left[S_n^* S_n\right] = N\sigma_n^2$. Thus, $\text{SNR}_{\text{post}} = 1/\sigma_n^2$, which implies

$$L = 1$$

The processing offers no gain beyond the pre-correlation SNR when the information symbols are completely unknown.

B. Known Data

Suppose $\{x_k\}_{k=1}^N$ is entirely known *a priori*. To maximize processing gain, we set $l_k = x_k$ for $k \in [1, N]$. Then

$$\begin{aligned}\mathbb{E}\left[S_x^* S_x\right] &= \mathbb{E}\left[\left(\sum_{k=1}^N x_k^* x_k\right)\left(\sum_{j=1}^N x_j^* x_j\right)\right] \\ &= \sum_{k=1}^N \sum_{j=1}^N \mathbb{E}\left[|x_k|^2 |x_j|^2\right] \\ &= \sum_{k=1}^N \mathbb{E}\left[|x_k|^4\right] + \sum_{\substack{i,j=1 \\ i\neq j}}^N \mathbb{E}\left[|x_i|^2\right]\mathbb{E}\left[|x_j|^2\right] \\ &= N\gamma + N(N-1)\end{aligned}$$

where $\gamma \triangleq \mathbb{E}\left[|x_k|^4\right]$ is the fourth moment of the modulation constellation normalized such that $\mathbb{E}\left[|x_k|^2\right] = 1$. For example, $\gamma = 1$ for constant-modulus constellations such as QPSK and 4QAM, whereas $\gamma = 33/25$ for 16QAM. Since $\mathbb{E}\left[S_n^* S_n\right] = N\sigma_n^2$, the processing gain becomes

$$L = \gamma + N - 1$$

For constant-modulus constellations ($\gamma = 1$), this reduces to $L = N$, corresponding to fully coherent accumulation across N symbols.

C. Partially Known Data

Suppose the transmitted symbols are not known exactly, but a prior PMF $p_k(x_k)$ for each $x_k \in \mathcal{C}$ is available. The local replica $\{l_k\}_{k=1}^N$ may be constructed in various ways, such as a maximum *a priori* replica

$$l_k = \operatorname*{argmax}_{x\in\mathcal{C}} p_k(x)$$

or a soft replica

$$l_k = \sum_{x\in\mathcal{C}} x\, p_k(x)$$

Without assuming a specific construction, define

$$\alpha \triangleq \mathbb{E}\left[|x_k|^2 |l_k|^2\right], \qquad \mu \triangleq \mathbb{E}\left[l_k^* x_k\right]$$

The signal component $\mathbb{E}\left[S_x^* S_x\right]$ becomes

$$\begin{aligned}\mathbb{E}\left[S_x^* S_x\right] &= \mathbb{E}\left[\left(\sum_{k=1}^N l_k^* x_k\right)\left(\sum_{j=1}^N x_j^* l_j\right)\right] \\ &= \sum_{k=1}^N \sum_{j=1}^N \mathbb{E}\left[l_k^* x_k x_j^* l_j\right] \\ &= \sum_{k=1}^N \mathbb{E}\left[|x_k|^2 |l_k|^2\right] + \sum_{k=1}^N \sum_{\substack{j=1 \\ j\neq k}}^N \mathbb{E}\left[l_k^* x_k x_j^* l_j\right]\end{aligned}$$

But

$$\sum_{k=1}^N \mathbb{E}\left[|x_k|^2 |l_k|^2\right] = N\alpha$$

and further calculation, exploiting independence of $l_k^* x_k$ from $x_j^* l_j$ for $k \neq j$, yields

$$\mathbb{E}\left[l_k^* x_k x_j^* l_j\right] = \mathbb{E}\left[l_k^* x_k\right] \mathbb{E}\left[x_j^* l_j\right] = \mu\, \mu^* = |\mu|^2$$

$$\sum_{k=1}^{N} \sum_{\substack{j=1 \\ j\neq k}}^{N} \mathbb{E}\left[l_k^* x_k x_j^* l_j\right] = N(N-1)|\mu|^2$$

Thus,

$$\mathbb{E}\left[S_x^* S_x\right] = N\alpha + N(N-1)|\mu|^2$$

Similarly, we compute

$$\begin{aligned}\mathbb{E}\left[S_n^* S_n\right] &= \mathbb{E}\left[\left(\sum_{k=1}^{N} l_k^* n_k\right)\left(\sum_{j=1}^{N} n_j^* l_j\right)\right] \\ &= \sum_{k=1}^{N}\sum_{j=1}^{N} \mathbb{E}\left[l_k^* n_k n_j^* l_j\right]\end{aligned}$$

For $k \neq j$, $\mathbb{E}\left[n_k n_j^*\right] = 0$, so only diagonal terms remain:

$$\mathbb{E}\left[S_n^* S_n\right] = \sum_{k=1}^{N} \mathbb{E}\left[|l_k|^2\right] \mathbb{E}\left[|n_k|^2\right] = \sum_{k=1}^{N} \mathbb{E}\left[|l_k|^2\right]\, \sigma_n^2$$

Define $\rho \triangleq \mathbb{E}\left[|l_k|^2\right]$. It follows that

$$\mathbb{E}\left[S_n^* S_n\right] = N\sigma_n^2 \rho$$

and

$$\begin{aligned}\mathrm{SNR}_{\text{post}} &= \frac{\mathbb{E}\left[S_x^* S_x\right]}{\mathbb{E}\left[S_n^* S_n\right]} \\ &= \frac{\alpha + (N-1)|\mu|^2}{\sigma_n^2 \rho}\end{aligned}$$

Thus

$$L = \frac{\alpha + (N-1)|\mu|^2}{\rho} \tag{1}$$

This general model for L interpolates smoothly between the cases where the data is completely unknown or fully known, parameterized by the expectations α, γ, and μ.

II. Capture Metadata

As discussed in Section IV of the main text, the signal models and structural properties reported in this work are supported by a large corpus of Starlink Ku-band captures. This appendix provides detailed metadata for that corpus. Table I lists the Space Vehicle Identifier (SVID), hardware version, number of frames, and capture date for each observation session, demonstrating the diversity of the analyzed dataset across multiple satellite generations and time periods. The 1009-frame dataset from STARLINK-31848 serves as the exemplar frames used for the detailed analysis presented in the main text.

TABLE I: Capture Metadata

SVID	Generation	Frames	Capture Date
STARLINK-1274	v1.0	450	Jan. 13, 2025
STARLINK-5128	v1.5	150	Jan. 22, 2025
STARLINK-6148	v1.5	142	May 17, 2025
STARLINK-6128	v1.5	1677	Jun. 19, 2025
STARLINK-31322	v2.0-mini	162	Oct. 23, 2024
STARLINK-31848	v2.0-mini	1009	Jan. 21, 2025
STARLINK-32427	v2.0-mini	90	May 17, 2025
STARLINK-32879	v2.0-mini	136	Jun. 5, 2025
STARLINK-30861	v2.0-mini	199	Jun. 5, 2025
STARLINK-30963	v2.0-mini	101	Jun. 17, 2025
STARLINK-31750	v2.0-mini	257	Jun. 19, 2025